Effect of a Magnetic Field on Mixed Convection Heat and Mass Transfer from a Vertical Wedge in a Porous Medium by an Integral Approach

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Abstract

This study uses the integral method to study the heat and mass transfer by combined natural and forced convection from a vertical wedge with constant wall temperature and concentration in porous media saturated with an electrically conducting fluid in the presence of a transverse magnetic field. The simple approximate solutions obtained by the integral method are found to be in reasonable agreement with the nonsimilar solutions. This study presents simple analytic expressions so that any practicing engineer can easily and rapidly calculate the local and average values of important heat and mass transfer characteristics. An increase in the wedge angle parameter tends to increase the local Nusselt number and the local Sherwood number while an increase in the magnetic parameter decreases the local Nusselt number and the local Sherwood number. The ratio of the thermal boundary layer thickness to the concentration boundary layer thickness decreases with the magnetic parameter and the wedge angle parameter.

Keywords: Heat and Mass Transfer, Mixed Convection, Vertical Wedge, Porous Medium, Magnetic Field, Integral Method

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I. Introduction

The problems of the natural convection heat and mass transfer in porous media saturated with fluids may be met in real world. There has been considerable interest in studying flows of mixed convection heat and mass transfer of Newtonian fluids in porous media. The applications are found in geothermal energy technology, petroleum recovery, and underground disposal of chemical and nuclear waste.


Nakayama and Hossain [11], and Singh and Queeny [12] have used the integral method to obtain the analytic solution of couple heat and mass transfer due to buoyancy along a vertical surface in a fluid saturated porous medium with constant wall temperature and concentration. Cheng [13] studied the effect of a magnetic field on the heat and mass transfer by natural convection near a vertical surface with constant wall temperature and concentration in a porous medium by the integral method. Cheng [14] used an integral approach for heat and mass transfer by natural convection from truncated cones in porous media with variable wall temperature and concentration. Cheng [15] obtained the integral solutions for hydromagnetic natural convection heat and mass transfer from vertical surfaces with power law variation in wall temperature and concentration in porous media. Cheng [16] studied the hydromagnetic mixed convection heat and mass transfer from a vertical surface in a saturated porous medium by an integral method.

The present work applies the integral method to study the problem of heat and mass transfer by mixed convection from a vertical wedge with constant wall temperature and concentration embedded in porous media saturated with an electrically conducting fluid under the influence of a transverse magnetic field. The applied transverse magnetic field is assumed to be uniform and the magnetic Reynolds number is so small that induced magnetic field can be neglected. Further, the external electric field is assumed to be zero and the electric field due to polarization of charges is negligible. The calculated results obtained herein are compared with the nonsimilar solutions calculated by previous studies to assess the accuracy of the present integral method. With the simple analytical solution, any practicing engineer can easily and rapidly obtain the Nusselt and Sherwood numbers for the hydromagnetic mixed convection heat and mass transfer from a vertical wedge with constant wall temperature and concentration in porous media.

II. Analysis

Consider the mixed convection boundary-layer flow along a vertical wedge with constant wall temperature
and concentration embedded in a porous medium saturated with an electrically conducting fluid subject to a uniform transverse magnetic field. The flow configuration and coordinate system are shown in Figure 1. The flow along the vertical wedge is assumed to be steady, two-dimensional, laminar, and incompressible. The surface of the vertical wedge is maintained at a constant temperature $T_w$ different from the porous medium temperature $T_\infty$ sufficiently far from the surface of the vertical wedge. Moreover, the concentration of a certain constituent in the solution that saturates the porous medium varies from a constant concentration $C_w$ on the fluid side of the vertical wedge to $C_\infty$ sufficiently far from the surface of the vertical wedge. All the fluid properties are assumed to be constant, except for density variations in the buoyancy term.

\begin{align}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \quad (1) \\
u = U_\infty + \frac{\rho g K \cos A}{\mu} \left[ \beta_T (T - T_\infty) + \beta_C (C - C_\infty) \right] \left( 1 + \frac{K\sigma B_0^3}{\mu} \right)^{-1} \quad (2) \\
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \alpha \frac{\partial^2 T}{\partial y^2} \quad (3) \\
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} &= D \frac{\partial^2 C}{\partial y^2} \quad (4)
\end{align}

In Eqs. (1)-(4), $A$ is the half-angle of the vertical wedge. The x-coordinate is measured from the leading edge of the vertical wedge and y-coordinate is measured normal to the surface of the vertical wedge. $u$ and $v$ are the volume-averaged velocity in the x and y directions, respectively, $T$ and $C$ are temperature and concentration, respectively. Properties $\mu$, $\rho$, $\sigma$, and $B_0$ are the solution viscosity, density, electrical conductivity, and magnetic induction, respectively. $K$ and $\varepsilon$ are the permeability and the porosity of the porous medium, respectively. $\beta_T$ and $\beta_C$ are the coefficient of thermal expansion and the coefficient of concentration expansion, respectively. The thermal diffusivity $\alpha$ is defined as the thermal conductivity of the fluid saturated porous medium, divided by the specific heat capacity of the fluid alone. The mass diffusivity $D$ is the diffusivity of the constituent of interest measured through the fluid-saturated porous medium. It should be noted that the external flow is at a velocity $U_\infty = Bx^4$, and the wedge angle parameter is $\lambda = A/(\pi - A)$, and $B$ is a prescribed constant. The associated boundary conditions are:

The appropriate boundary conditions are:

\begin{align}
y = 0; \quad T = T_w, \quad C = C_w, \quad v = 0
\end{align}
\[ y \to \infty; \quad T \to T_w, \quad C \to C_w, \quad u \to U_w \]  
\[ (6) \]

Integrating Eqs. (3) and (4) about \( y \) from 0 to \( \infty \) and using Eq. (1), we obtain the following integral equations

\[ \frac{d}{dy} \int_0^y (u \theta) dy = -\alpha \frac{\partial \theta}{\partial y} \bigg|_{y=0} \]  
\[ (7) \]

\[ \frac{d}{dy} \int_0^y (u \phi) dy = -D \frac{\partial \phi}{\partial y} \bigg|_{y=0} \]  
\[ (8) \]

where \( \theta = \frac{T - T_w}{T_w - T_o} \) and \( \phi = \frac{C - C_w}{C_w - C_o} \).

To satisfy the boundary conditions, Eqs. (5) and (6), the profiles of dimensionless temperature and concentration is assumed to be the following functions:

\[ \theta = 1 - 2 \frac{y}{\delta_t} + 2 \left( \frac{y}{\delta_t} \right)^3 - \left( \frac{y}{\delta_t} \right)^4 \]  
\[ (9) \]

\[ \phi = 1 - 2 \frac{y}{\delta_c} + 2 \left( \frac{y}{\delta_c} \right)^3 - \left( \frac{y}{\delta_c} \right)^4 \]  
\[ (10) \]

It should be noted that the temperature and concentration profile functions defined in Eqs. (9) and (10) also satisfy the compatibility conditions and the smoothness conditions:

\[ y = 0; \quad \frac{\partial^2 \theta}{\partial y^2} = 0, \quad \frac{\partial^2 \phi}{\partial y^2} = 0 \]  
\[ (11) \]

\[ y \to \infty; \quad \frac{\partial \theta}{\partial y} \to 0, \quad \frac{\partial \phi}{\partial y} \to 0, \quad \frac{\partial^2 \theta}{\partial y^2} \to 0, \quad \frac{\partial^2 \phi}{\partial y^2} \to 0 \]  
\[ (12) \]

Equation (11) is obtained by evaluating Eqs. (3) and (4) at \( y = 0 \). Using Eqs. (2), (9) and (10), and integrating Eqs. (7) and (8) about \( y \) from 0 to \( \infty \), we can get two ordinary differential equations for the thermal boundary-layer thickness \( \delta_t \) and the concentration boundary-layer thickness \( \delta_c \). The solutions of two ordinary differential equations can be given by

\[ \delta_t^* = \delta_t \frac{x}{\sqrt{Ra + \sqrt{Pe}}} \]  
\[ (13) \]

\[ \delta_c^* = \delta_c \frac{x}{\sqrt{Ra + \sqrt{Pe}}} \]  
\[ (14) \]

In Eqs. (13)-(14), \( Pe = u_w x / \alpha \) is the Peclet number and \( Ra = \frac{K D \beta g \cos A(T_w - T_o)}{(\alpha u) x} \) as the Darcy-modified Rayleigh number, and the unknowns \( \delta_t^* \) and \( \delta_c^* \) can be given by

\[ \delta_t^* = 2 \left[ (1 + M^2) \right]^{1/2} \left[ 0.3(1 + 2\lambda) \xi^2 + (1 + M^2) \right]^{1/2} \left[ 0.1825 + NF(\Delta) \right] \left[ 1 - \xi^2 \right]^{1/2} \]  
\[ (15) \]

\[ \delta_c^* = 2 Le^{-1/2} \left[ 0.3(1 + 2\lambda) \xi^2 + (1 + M^2) \right]^{1/2} \left[ 0.1825 N + \DeltaF(\Delta) \right] \left[ 1 - \xi^2 \right]^{1/2} \]  
\[ (16) \]

Moreover, in Eqs. (15)-(16), \( \xi = 1 / \left[ 1 + (Ra/Pe)^{1/2} \right] \) is the mixed convection parameter. It is noted that
\( \xi = 0 \) and \( \xi = 1 \) correspond to pure natural convection and pure forced convection cases, respectively. Moreover, \( \Delta = \delta_i / \delta_e \) is used to represent the ratio of the boundary layer thickness, \( Le = \alpha / D \) is the Lewis number, \( N = \beta_i (C_u - C_e) / \beta_e (T_e - T_w) \) is the buoyancy ratio, \( M^2 = (KnB^2_e)/(\epsilon a) \) is the square of the magnetic parameter, and the function \( F(A) \) can be expressed as

\[
F(A) = \frac{3}{10} - \frac{2}{15} \Delta + \frac{3}{140} \Delta^3 - \frac{1}{180} \Delta^4 \quad \text{for} \quad A < 1
\]

\[
F(A) = \frac{3}{10A} - \frac{2}{15A^2} + \frac{3}{140A^3} - \frac{1}{180A^4} \quad \text{for} \quad A > 1
\]  

Results of practical interest are rates of heat and mass transfer from the wall to the fluid. The local Nusselt number and the local Sherwood number are given by

\[
Nu = \frac{2}{\delta_e} \left( \sqrt{Pe} + \sqrt{Ra} \right)
\]

\[
Sh = \frac{2}{\delta_e} \left( \sqrt{Pe} + \sqrt{Ra} \right)
\]

In Eqs. (19) and (20) \( Nu = h_x / k \) and \( Sh = h_m x / D \), where \( h \) and \( h_m \) are the local heat transfer coefficient and the local mass transfer coefficient, respectively.

### III. Results and Discussion

In Table 1, we compare the present results for the local Nusselt number with those obtained by Yih [8] for mixed convection heat transfer from a wedge with constant wall temperature embedded in a porous medium. The agreement between our results and the exact values is within 8.53%. We also compare the results for the local Nusselt number and the local Sherwood number calculated in this work with those obtained by Yih [6] for mixed convection heat and mass transfer from a wedge with constant wall temperature and concentration in a porous medium, as shown in Table 2. The agreement between the present results and the exact values is within 8.3 %.

**Table 1  Comparison of the local Nusselt number for various values of mixed convection parameters with \( M = 0, \ Le = 1, \ \lambda = 1/3 \) and \( N = 0 \).**

<table>
<thead>
<tr>
<th>( \xi )</th>
<th>Exact (a)</th>
<th>Present (b)</th>
<th>Relative error ((b-a)/a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yih [8]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.4437</td>
<td>0.4272</td>
<td>-0.03719</td>
</tr>
<tr>
<td>0.1</td>
<td>0.4044</td>
<td>0.3910</td>
<td>-0.03314</td>
</tr>
<tr>
<td>0.2</td>
<td>0.3769</td>
<td>0.3699</td>
<td>-0.01857</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3643</td>
<td>0.3667</td>
<td>0.00659</td>
</tr>
<tr>
<td>0.4</td>
<td>0.3686</td>
<td>0.3817</td>
<td>0.03553</td>
</tr>
<tr>
<td>0.5</td>
<td>0.3900</td>
<td>0.4131</td>
<td>0.05923</td>
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<tr>
<td>0.6</td>
<td>0.4261</td>
<td>0.4574</td>
<td>0.07346</td>
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<tr>
<td>0.7</td>
<td>0.4731</td>
<td>0.5113</td>
<td>0.08074</td>
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<tr>
<td>0.8</td>
<td>0.5278</td>
<td>0.5721</td>
<td>0.08393</td>
</tr>
<tr>
<td>0.9</td>
<td>0.5878</td>
<td>0.6378</td>
<td>0.08506</td>
</tr>
<tr>
<td>1.0</td>
<td>0.6515</td>
<td>0.7071</td>
<td>0.08534</td>
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</table>
Table 2  Comparison of local Nusselt and local Sherwood numbers for various values of Lewis numbers and mixed convection parameters with \( \lambda = 0, \ N = 1 \) and \( M = 0 \).

<table>
<thead>
<tr>
<th>Le ( \xi )</th>
<th>( \frac{Nu}{(\sqrt{Pe} + \sqrt{Ra})} )</th>
<th>( \frac{Sh}{(\sqrt{Ra} + \sqrt{Pe})} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact (a)</td>
<td>Present (b)</td>
<td>( (b - a)/a )</td>
</tr>
<tr>
<td>0.01</td>
<td>0</td>
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<td>0.5642</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.6276</td>
</tr>
<tr>
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<td>0.2</td>
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</tr>
<tr>
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<tr>
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<td>0</td>
<td>0.4700</td>
</tr>
<tr>
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<td>0.4618</td>
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<tr>
<td></td>
<td>1</td>
<td>0.5642</td>
</tr>
</tbody>
</table>

Figure 2 shows the variation of the local Nusselt number \( \frac{Nu}{(\sqrt{Pe} + \sqrt{Ra})} \) with the mixed convection parameter \( \xi \) for different values of magnetic parameter \( M \). As the mixed convection parameter \( \xi \) increases from 0 (pure natural convection) to 1 (pure forced convection), the value of \( \frac{Nu}{(\sqrt{Pe} + \sqrt{Ra})} \) decreases at low \( \xi \) values, reaches a minimum, and then increases. Moreover, increasing the magnetic parameter \( M \) tends to decrease the local Nusselt number \( \frac{Nu}{(\sqrt{Pe} + \sqrt{Ra})} \). That is because increasing the magnetic parameter \( M \) retards the flow, thus decreasing the heat transfer between the fluid and the wall.

![Figure 2](image_url)
Figure 3 depicts the local Sherwood number $\frac{Sh}{\left(\sqrt{Pe} + \sqrt{Ra}\right)}$ as a function of the mixed convection parameter $\xi$ for various values of the magnetic parameter $M$. As the mixed convection parameter $\xi$ increases from 0 (pure natural convection) to 1 (pure forced convection), the value of $\frac{Sh}{\left(\sqrt{Pe} + \sqrt{Ra}\right)}$ decreases at low $\xi$ values, reaches a minimum, and then increases. Moreover, an increase in the magnetic parameter $M$ decreases the local Sherwood number $\frac{Sh}{\left(\sqrt{Pe} + \sqrt{Ra}\right)}$. That is because increasing the magnetic parameter $M$ tends to decrease the fluid flow velocity and thus reduces the mass transfer near the wall. This behavior is much pronounced for the natural-convection dominated flows.

![Figure 3](image1)

**Fig. 3** Effects of magnetic parameters on the local Sherwood number.

The boundary layer thickness ratio $\Delta$ is plotted in Figure 4 as a function of the mixed convection parameter $\xi$ for various values of the magnetic parameter $M$. It is clearly shown in this figure that the boundary layer thickness ratio $\Delta$ decreases with an increase in the mixed convection parameter $\xi$ and the magnetic parameter $M$. Figure 5 shows the local Nusselt number $\frac{Nu}{\left(\sqrt{Pe} + \sqrt{Ra}\right)}$ as a function of the mixed convection parameter $\xi$ for various values of the wedge angle parameter $\lambda$. Increasing the wedge angle parameter $\lambda$ tends to increase the local Nusselt number $\frac{Nu}{\left(\sqrt{Pe} + \sqrt{Ra}\right)}$, and the phenomenon is more evident in forced convection-dominated flows than in natural-convection dominated flows.

![Figure 4](image2)

**Fig. 4** Effects of magnetic parameters on the boundary layer thickness ratio.
Fig. 5  Effects of wedge angle parameters on the local Nusselt number.

Fig. 6  Effects of wedge angle parameters on the local Sherwood number.

Fig. 7  Effects of wedge angle parameters on the boundary layer thickness ratio.
The local Sherwood number $Sh/\left(\sqrt{Pe}+\sqrt{Ra}\right)$ is plotted as a function of the mixed convection parameter $\xi$ for various values of the wedge angle parameter $\lambda$ in Figure 6. It is clearly shown that the local Sherwood number $Sh/\left(\sqrt{Pe}+\sqrt{Ra}\right)$ increases as the wedge angle parameter $\lambda$ is increased. That is due to the fact that an increase in wedge angle parameters $\lambda$ leads to an increase in the flow velocity and thus increasing the mass transfer between the surface and the fluid, and the phenomenon is more pronounced in forced convection-dominated flows than in natural-convection dominated flows. Figure 7 shows the effect of the wedge angle parameter $\lambda$ on the boundary layer thickness ratio $\Delta$. Increasing the wedge angle parameter tends to decrease the boundary layer thickness ratio.

Figure 8 plots the local Nusselt number $Nu/\left(\sqrt{Pe}+\sqrt{Ra}\right)$ as a function of the mixed convection parameter $\xi$ for various values of the Lewis number $Le$. The local Nusselt number $Nu/\left(\sqrt{Pe}+\sqrt{Ra}\right)$ tends to decrease as the Lewis number $Le$ is increased, and this phenomenon is much pronounced in the free-convection dominated flows. The variation of the local Sherwood number $Sh/\left(\sqrt{Pe}+\sqrt{Ra}\right)$ with the mixed convection parameter $\xi$ for various values of the Lewis number $Le$ is shown in Figure 9. The local Sherwood number $Sh/\left(\sqrt{Pe}+\sqrt{Ra}\right)$ tends to increase as the Lewis number $Le$ is increased.

**Fig. 8** Effects of Lewis numbers on the local Nusselt number.

**Fig. 9** Effects of Lewis numbers on the local Sherwood number.
The effect of the Lewis number $Le$ on the boundary layer thickness ratio $\Delta$ is depicted in Figure 10. It is clearly shown in this figure that the boundary layer thickness ratio $\Delta$ increases with an increase in the Lewis number $Le$. Figure 11 plots the local Nusselt number $Nu/(Pe^{0.5} + Ra^{0.5})$ as a function of the mixed convection parameter $\xi$ for various values of the buoyancy ratio $N$. The local Nusselt number $Nu/(Pe^{0.5} + Ra^{0.5})$ tends to increase as the buoyancy ratio $N$ is increased in the natural convection dominated flows, because increasing buoyancy ratio tends to accelerate the flow, thinning the thermal boundary layer and thus increasing the temperature gradients at the wall.

![Fig. 10  Effects of Lewis numbers on the boundary layer thickness ratio.](image)

![Fig. 11  Effects of buoyancy ratios on the local Nusselt number.](image)

The variation of the local Sherwood number $Sh/(Fe^{0.5} + Ra^{0.5})$ with the mixed convection parameter $\xi$ for various values of the buoyancy ratio $N$ is plotted in Figure 12. The local Sherwood number $Sh/(Fe^{0.5} + Ra^{0.5})$ tends to increase as the buoyancy ratio $N$ is increased in the natural convection dominated flows. That is because increasing the buoyancy ratio enhances the buoyancy force, thinning the concentration boundary layer and thus increasing the mass transfer near the wall. Figure 13 shows the effect of the buoyancy ratio $N$ on the boundary layer thickness ratio $\Delta$. The boundary layer thickness ratio $\Delta$ increases with an increase in the buoyancy ratio $N$ in the natural convection dominated flows.
IV. Conclusions

An integral approach has been used to study the mixed convection heat and mass transfer from a vertical wedge with constant wall temperature and concentration embedded in porous media saturated with an electrically conducting fluid in the presence of a transverse magnetic field. The approximate solutions obtained from the integral method are in reasonable agreement with the exact solutions. This study presents simple analytic expressions for engineers to easily and rapidly calculate the physical characteristics of the heat and mass transfer. Increasing the wedge angle parameter increases the local Nusselt number and the local Sherwood number while increasing the magnetic parameter decreases the local Nusselt number and the local Sherwood number. The ratio of the thermal boundary layer thickness to the concentration boundary layer thickness decreases with the magnetic parameter and wedge angle parameter.
References


